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Thermodynamic Analysis of the Influence of Electric Fields on Frost Formation

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Nomenclature

A = surface area
a = height of the needle frost
b = radius of the needle frost
E = average electric field
E_e = electric field at the tip
F = Helmholtz free energy
f = driving force of sublimation
G = Gibbs free energy
g = specific Gibbs free energy
K_{ve} = electric field coefficient
m = mass
p = saturation pressure
p^* = pressure of water vapor near the frost surface
R = universal gas constant
r_c = critical radius of ice nucleation
T = temperature or saturation temperature
γ = interfacial energy of unit area defined by Eq. (9)
ε_0 = dielectric constant
λ = latent heat
ρ = density
χ = polarization rate
ω = specific electric energy

Subscripts
α = solid phase
β = vapor phase
σ = interface phase
0 = relative or reference point
∞ = no electric field

Introduction

THE typical frost formation process in the absence of electrostatic fields has been described in detail by Hayashi et al.[1] In the initial stage of frost formation on a horizontal surface, a thin frost is formed, covering the cooled surface. Frost crystals, which are relatively far apart, then form on this thin frost layer. These frost crystals grow in a vertical direction, all at approximately the same rate, forming a structure of long thin vertical shafts, similar to a forest of trees. This rough frost formation continues to grow through the formation of branches around the top of the frost crystals or through the interaction of each crystal, and gradually forms a relatively flat surface as shown in Fig. 1a.

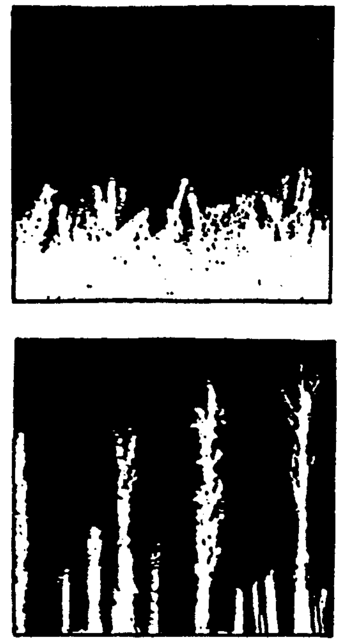

Fig. 1 Schematic of frost formation: a) with and b) without electrostatic fields.

The formation of frost in a uniform electrostatic field is quite different in several respects.[2,3] Most notable is that in the initial frost nucleation, the frost does not appear to form evenly on the cooled surface, but rather forms in pits or small depressions. When the first frost nucleations spread and increase in size, more new frost nucleations are formed on the surface. After a short period of time, small needle-like crystals of frost suddenly appear at the frost nucleation sites. The rate of growth of these needles is quite rapid and not particularly uniform, however, the growth for all of these needles is in the vertical direction, perpendicular to the cooled surface, as shown in Fig. 1b. This behavior has a number of similarities to the formation of liquid droplets occurring in dropwise condensation.

Because little is known about the behavior of these frost needles, and the physical processes involved in the frost growth in the presence of electrostatic fields, difficulties have been encountered in predicting the effects these fields have on the frost formation. Until recently, no satisfactory results, either qualitative or quantitative, have been available. In an effort to explain qualitatively some of the experimentally observed phenomenon, the fundamental thermodynamic expressions that describe the effect electrostatic fields have on the vapor–solid phase change processes occurring during frost formation, have been developed.

Theoretical Analysis

Frost formation through sublimation is a process of ice-crystal growth in the gaseous phase. When a surface at a temperature below the freezing temperature (0°C at atmospheric pressure) comes into contact with moist air having a dew-point temperature greater than the temperature of the surface, frost will form. Because the oversaturated water vapor is a metastable phase in which the Gibbs free energy is higher than that of an ice crystal in steady state, ice crystals can form (even under steady-state conditions). Assuming that the water vapor behaves as an ideal gas, the driving force of sublimation may be expressed as

$$f = RT \ln(P^*/P) \tag{1}$$

As a result of the oversaturated vapor pressure of the water

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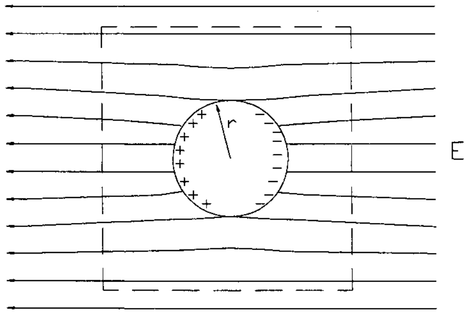

Fig. 2 Schematic of vapor–solid phase change of the ice ball under electric fields.

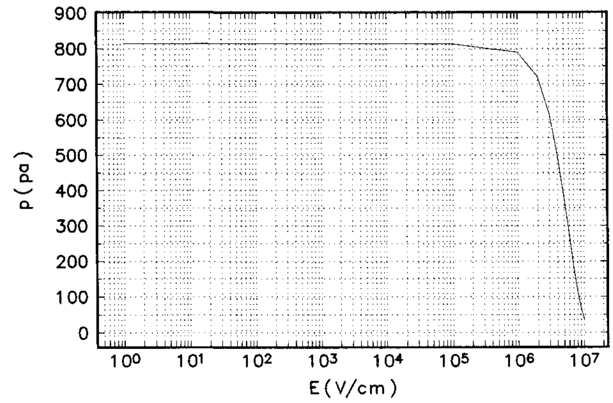

Fig. 3 Relationship between the saturated pressure and the electric field (T = 273.15 K, ε_0 = 8.845 × 10^{-12} C^2/N, m^2, ρ_α = 914.7 kg/m^3, χ = 80).

vapor at or near the frost layer, the ice-crystal grows, moving the interface outward and into the vapor phase.

If a homogeneous ice crystal or ball with no electric charge is placed into a uniform electrostatic field filled with moist or humid air, the ice crystal or ball will be polarized as shown in Fig. 2. Assuming a closed thermodynamic system consisting of the ice crystal, the humid air, and the vapor–solid interface, all within a uniform electrostatic field, the total Gibbs free energy can be written as

$$\begin{aligned} G &= G_\alpha + G_\beta + G_\sigma \\ &= g_\alpha m_\alpha + \omega_\alpha m_\alpha + g_\beta m_\beta + \omega_\beta m_\beta + G_\sigma \end{aligned} \tag{2}$$

Given a system at constant temperature and pressure, in which some change of state of a unit mass dm from vapor to ice occurs, equilibrium requires that the Gibbs free energy be at a minimum, i.e.,

$$\mathrm{d}G = \mathrm{d}G_\alpha + \mathrm{d}G_\beta + \mathrm{d}G_\sigma = 0 \tag{3}$$

Evaluating the Gibbs free energy for each phase at an equilibrium state considering the total mass of water being constant during the phase change process of this system, and using the expression[4] $\omega_\alpha = -\chi\varepsilon_0E^2/(2\rho_\alpha)$ for the electric energy of a unit mass of the ice crystal, the following relationship can be obtained

$$g_\beta - g_\alpha = \frac{\mathrm{d}G_\sigma}{\mathrm{d}m_\alpha} - \frac{\chi\varepsilon_0E^2}{2\rho_\alpha} \tag{4}$$

Neglecting the interface energy influence and assuming that the electrostatic field is equal to zero, Eq. (4) becomes

$$(g_\beta)_\infty = g_\alpha \tag{5}$$

and substituting Eq. (5) into Eq. (4) yields

$$g_\beta - (g_\beta)_\infty = \frac{\mathrm{d}G_\sigma}{\mathrm{d}m_\alpha} - \frac{\chi\varepsilon_0E^2}{2\rho_\alpha} \tag{6}$$

Table 1 Dimensions of needle frost with the electric field and saturation pressure

$a/2b$[a]	E_e/E[b]	p/p_∞[c]
1.00	5.76	1.0000
2.00	13.26	0.9999
4.00	35.18	0.9991
6.00	65.37	0.9967
8.00	103.17	0.9919
10.00	148.17	0.9833
12.00	200.07	0.9698
14.00	258.62	0.9500
16.00	323.67	0.9228
18.00	394.98	0.8872
20.00	472.74	0.8424
22.00	556.36	0.7886
24.00	646.00	0.7260
26.00	741.42	0.6559
28.00	842.75	0.5799
30.00	950.25	0.5002

[a]Aspect ratio. [b]Electric field ratio. [c]Saturation pressure ratio.

From Eq. (6), it can be seen that variations in electrostatic fields can result in changes in the driving force of the phase change necessary for the frost phase change to become smaller. This implies that the trend of the phase change is from the vapor state to the solid state, making it easier to form ice nuclei on the cooled surface. The interface energy influence on the driving force of phase change, however, is opposite that caused by the electrostatic field.

Assuming the vapor behaves as an ideal gas at constant temperature, the saturation pressure with the electrostatic field influence can be determined from the definition of Gibbs free energy, and shown to be

$$p = p_0 \exp\left[\frac{\lambda}{R}\left(\frac{T - T_0}{TT_0}\right) + \frac{1}{RT}\left(\frac{\mathrm{d}G_0}{\mathrm{d}m_\alpha} - \frac{\chi\varepsilon_0E^2}{2\rho_\alpha}\right)\right] \tag{7}$$

It is apparent from this expression that increasing the electrostatic field, decreases the saturation pressure. Figure 3 illustrates the relationship between the saturation pressure and the electrostatic field as indicated by Eq. (7). As shown, neglecting the interface energy influence at a given temperature, the saturation pressure decreases when the electrostatic field increases. If the electrostatic field is allowed to approach zero, Eq. (7) will reduce to the Clapeyron equation, clarifying the physical significance of this expression.

Discussion of Results

Babakin et al.[2] and Meng et al.[3] found that in a uniform electrostatic field, "needle" frost was formed on cooled or cryo surfaces and that the type and growth of the frost layer depended not only on the humid air temperature, the pressure, relative humidity, air speed, and surface temperature, but also on the electrostatic field influence and the polarization characteristics of water. Because of the large number of parameters, these investigators were unable to explain the phenomena behind the formation of "needle frost" on cooled surfaces under the influence of an electrostatic field. However, from the perspective of phase change dynamics,[5] the frost formation from the vapor phase to the solid phase results primarily from the driving force of the phase change. The direction of the strong or weak forces of an electrostatic field, the polarization characteristics of water molecules, the cooled surface conditions, and the environmental conditions can all affect the driving force of the phase change, causing it to vary at different locations on the surface. From Eq. (1), it is clear that the driving force depends primarily on the vapor pressure and the saturation pressure, but the presence of the electrostatic field can change these pressures. From Fig. 3, it is apparent that when the electrostatic field is larger than 10^5 V/cm, the electrostatic field significantly influences the saturation pressure, but the actual electrostatic field applied by an electrostatic source is only 2000–8000 V/cm^3. It is a well-known fact that for a uniform electrostatic field, the electric field at the tip of any rounded object inserted into the field increases significantly as the inverse of the radius, $1/r_e$, increases, where r_e is the radius of the end of the object. Babakin et al.[2] studied the relationship between the electric field and the needle frost experimentally, and found

$$K_{yc} = 1 + \frac{1}{\frac{1}{2}\ln[(\eta_0 + 1)/(\eta_0 - 1)] - (1/\eta_0)} \times \left[\frac{1}{(a/c) - (c/a)} - \frac{1}{2}\ln\frac{(a/c) + 1}{(a/c) - 1}\right] \qquad (8)$$

where K_{yc} is the electric field coefficient, which is equal to the ratio of the electric field at the sharp point of the needle frost to the average uniform electric field that is applied, a is the height of the needle frost, $2b$ is the diameter of the needle frost, c is equal to $(a^2 - b^2)^{0.5}$, and η_0 is the ratio a/c. The resulting electric field can be determined from Eq. (8), and then using Eq. (7) the corresponding saturation pressure can be found. These results are listed in Table 1 where the average electric field E is assumed to be 5000 V/cm.

During the initial stage of frost formation, the frost grows very slowly both with and without an electric field. As the crystal branch grows, a sharp point appears on the end of the crystal, and as illustrated in Table 1 or Eq. (8), the electric field will increase. From Eq. (7) the saturation pressure will decrease, and the driving force of the phase change from Eq. (1) will increase significantly. Therefore, the growth of the frost crystal is accelerated. Since the gradient of the electric field is largest along the direction of the electric field, the driving force of the phase change is also the largest in this direction, and the frost grows more quickly along this axis. With the growth of frost, the ratio of the height of the frost to the diameter becomes larger, and as shown in Table 1, the electric field and the driving force of the phase change increase further, promoting continued increases in the height of the frost formation. Once the frost is formed, it will grow very rapidly along the direction of the electric field, and within a very short time will approach the long slender shape of the characteristic needle frost.

Clearly, the electrostatic field changes the saturation pressure and the driving force of the phase change, resulting in the needle frost formation. This explanation also qualitatively verifies the experimental conclusion that when the electrostatic field increases, the condensing temperature increases.[6] This also brings into question the previously offered explanation that the electric field at the surface of the crystals might be sufficient to modify the mean surface migration distance of water molecules, and hence, change the process of ice formation and growth.[6] This explanation cannot satisfactorily explain the entire growth process of ice needles under the influence of electric fields.

In the process of deriving Eq. (7), the interface energy influence is included in the term $\mathrm{d}G_\sigma/\mathrm{d}m_\alpha$. Since the interface thickness is very thin and only several molecule layers thick, the Gibbs free energy is nearly equal to the interface Helmholtz energy. Following the definition of the interface free energy of vapor–liquid[7]

$$\gamma = \frac{\mathrm{d}F_\sigma}{\mathrm{d}A_\sigma} \quad \text{or} \quad \mathrm{d}F_\sigma = \gamma\,\mathrm{d}A_\sigma = \gamma 8\pi r\,\mathrm{d}r \qquad (9)$$

and considering the mass change of the ice ball, the critical radius of ice nucleation can be obtained as

$$r_c = \frac{2\gamma}{\rho_\alpha RT\ln\dfrac{p}{p_0} - \dfrac{\lambda\rho_\alpha}{R}\left(\dfrac{T - T_0}{T_0}\right) + \dfrac{\chi\varepsilon_0 E^2}{2}} \qquad (10)$$

From Eq. (10), it can be shown that the critical radius of ice nucleation under the influence of an electrostatic field is smaller than when no electrostatic field is present, making it easier to form ice nuclei in the presence of electrostatic fields.

Conclusions

The fundamental thermodynamic expressions that describe the effect of electrostatic fields on the vapor–solid phase change processes occurring during frost formation have been derived in an effort to qualitatively explain some of the experimentally observed phenomenon occurring in this process. Utilizing fundamental thermodynamic principles and phase-change theory, a saturation pressure equation and an expression for the critical radius of ice nuclei under an electrostatic field were derived.

The results indicate that when the electric field increases, the saturation pressure at the surface of the needle frost and the critical radius of ice nucleation both decrease. In addition, from the perspective of phase-change dynamics, since the saturation pressure will decrease with increases in the electrostatic field, the driving force of the phase change process increases significantly and results in the needle frost formation on cooled or cryo surfaces under the influence of electrostatic fields.

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Mixed Convection in a Vertical Channel with Heated Blocks

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Introduction

ALTHOUGH natural convection in heated vertical channels has been studied extensively,[1] theoretical study on mixed convection has been essentially limited to the buoyancy-aided flow.[2] Recently, Gau et al.[3] studied both buoyancy-aided and buoyancy-opposed convection in flat-plate channels. Kim and Boehm[4] considered the buoyancy-aided

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